Fluid Mechanics FE Review

I'm so sorry I had to cancel tonight's session,

Please look through these notes and review problems. I will be happy to answer any questions you have and meet with you individually if you would like. If there is enough interest, I will be happy to schedule a makeup session.

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Fluid Mechanics FE Review

These slides contain some notes, thoughts about what to study, and some practice problems. The answers to the problems are given in the last slide.

In the review session, we will be working some of these problems. Feel free to come to the session, or work the problems on your own. I am happy to answer your email questions or those that you bring to the session.

Good luck!

Fluid Mechanics FE Review

MAJOR TOPICS

Fluid Properties

Fluid Statics

Fluid Dynamics

Fluid Measurements

Dimensional Anaysis

**Most equations and problems taken from Professional Publications, Inc. FERC Review Course Book

Fluid Mechanics FE Review

It will be very helpful to memorize the following concepts and equations:

- Specific weight, density, and specific gravity (γ, ρ, SG) $\gamma = \rho g$ $S.G. = \frac{\gamma_{fluid}}{\gamma_{H_2O}} = \frac{\rho_{fluid}}{\rho_{H_2O}}$
- Hydrostatics pressure equation / manometry $P = \gamma h$
- Force magnitude and location due to hydrostatic pressure for horizontal and vertical plane walls
- Conservation of mass / continuity $\Sigma \dot{m}_{in} = \Sigma \dot{m}_{out}$
- Conservation of energy / Bernoulli and Energy Eqn $\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2$
- Darcy Eqn $h_{L,\text{due to friction}} = f \frac{L}{D} \frac{v^2}{2g}$
- Relative roughness equation ε/D
- Drag equation $F_D = \frac{1}{2} C_D A \rho v^2$
- How to use the Moody Diagram

Fluid Mechanics

Properties

Property	Symbol & Equation	Definition	Etc.
Density	$\rho = \frac{m}{\forall}$	$\frac{mass}{volume}$	
Specific Weight	$\gamma = \rho g$	*density x gravity*	
Specific Gravity	$SG = \frac{\rho_x}{\rho_{water}} = \frac{\gamma_x}{\gamma_{water}}$		
Viscosity	$\mu = \frac{\tau}{du/dy}$	$\frac{shear\ stress}{velocity\ gradient}$	
Kinematic viscosity	$v = \frac{\mu}{\rho}$	$\frac{viscosity}{density}$	
Ideal Gas Law	$p = \rho R_{gas} T$	*Use to find properties of gasses*	$R_{gas} = \frac{\bar{R}}{molec.\,wt.}$

Make sure you know the relationship between density, specific weight, and specific gravity!

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Fluid Mechanics

Properties

Fluids

- Substances in either the liquid or gas phase
- Cannot support shear

$\boxed{\text{FLUID PROPERTIES}}$

DENSITY, ρ

SPECIFIC (UNIT) WT, $\gamma = \rho g$

SPECIFIC GRAVITY, $SG = \frac{\rho}{\rho_{H_2O}} = \frac{\gamma}{\gamma_{H_2O}}$

SURFACE TENSION, $\sigma = \frac{\text{Force}}{\text{Length}}$

$\sigma = \frac{F}{2L}$ (WIRE FRAME) $= \frac{F}{4\pi r}$ (DU NOUY RING)

VAPOR PRESSURE, P_v ; VISCOSITY, μ (ABSOLUTE OR DYNAMIC) ; KINEMATIC VISCOCITY, $\nu = \mu/\rho$

- MAKE SURE YOU USE THE PROPER VALUES ACCORDING TO H_2O TEMPERATURE!

IDEAL GAS LAW: P=pressure; T=TEMP.; ρ= DENSITY; R= GAS CONSTANT

$p\forall = P/\rho = RT$ (volume; density), $R = \bar{R}/MW$ (Molecular wt.), $\bar{R} = 8.314$ $kJ/(kmol\ °K)$; $\frac{P_1 \forall_1}{T_1} = \frac{P_2 \forall_2}{T_2}$

Make sure you use the ideal gas law when calculating properties for gasses!!

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Mass & Weight

1. 10.0 L of an incompressible liquid exert a force of 20 N at the earth's surface. What force would 2.3 L of this liquid exert on the surface of the moon? The gravitational acceleration on the surface of the moon is 1.67 m/s^2

(A) 0.39 N
(B) 0.78 N
(C) 3.4 N
(D) 4.6 N

$F = ma \Rightarrow m = \frac{F}{A} = \frac{20\,N}{9.81\,m/s^2} = 2.04\,kg$

$\rho = \frac{m}{V} = \frac{2.04\,kg}{10\,L} = 0.204\,kg/L$

on the moon:

$F = ma = \rho V A$

$= 0.204\,kg/L\,(2.3\,L)(1.67\,m/s^2) = 0.784\,N$

Fluid Mechanics – Fluid Properties

Property	Symbol & Equation	Definition	Etc.
Vapor Pressure	p_v	Pressure at which liquid and vapor are in equilibrium	Used to predict cavitation (local pressure < vapor pressure
Bulk Modulus	$\rho = \frac{m}{V}$	$\frac{mass}{volume}$	
Speed of Sound	$c, liquid = \sqrt{k/\rho}$ $c, gas = \sqrt{krt}$	*Velocity of propagation of a small wave*	k = Bulk modulus $k_{air} = 1.4$

SPEED OF SOUND

Function of Bulk Modulus (which correlates to compressibility)

FOR IDEAL GAS: $c = \sqrt{kRT}$ (T must be in K or °R)

k = ratio of specific heats

MACH NUMBER $M = \frac{v}{c} = \frac{\text{object velocity}}{\text{speed of sound}}$

Mach Number

2. A jet aircraft is flying at a speed o f 1700 km/h. The air temperature is 20°C. The molecular weight of air is 29 g/mol. What is the Mach number of the aircraft?

(A) 0.979
(B) 1.38
(C) 1.92
(D) 5.28

Basically plug & chug

Eq's are in the manual: $R = \frac{\bar{R}}{M.W} = \frac{\text{universal gas const.}}{\text{molec. wt.}}$

speed of sound, $c = \sqrt{kRT}$

k look up → = 1.4

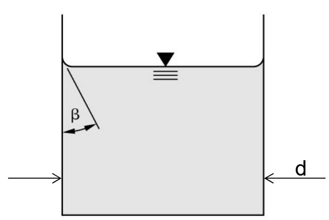
$$c = \sqrt{\frac{k\bar{R}T}{(MW)_{air}}} = \sqrt{\frac{1.4(8.314 \ \text{J/kmol}\cdot\text{K})(20°\text{C}+273.15)\text{K}}{29 \ \text{kg/kmol}}}$$

$c = 343$ m/s

$$\text{Mach \#}, M = \frac{V}{c} = \frac{1700 \ \text{km/hr}\,(1000 \ \text{m/km})}{343 \ \text{m/s}\left(\frac{3600 \ \text{s}}{\text{hr}}\right)} = \underline{\underline{1.38}}$$

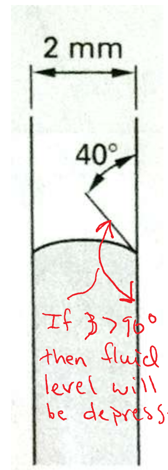
Fluid Mechanics – Fluid Properties

Property	Symbol & Equation	Definition	Etc.
Surface Tension	$\sigma = \frac{F}{L}$	$\frac{force}{length}$	
Capillary Rise	$h = \frac{4\sigma \cos\beta}{\rho d_{tube} g}$	*Distance a liquid will rise (or fall) in a "tube"*	



Surface Tension

3. A 2 mm (inside diameter) glass tube is placed in a container of mercury. An angle of 40° is measured as illustrated. The density and surface tension of mercury are 13550 kg/m^3 and 37.5 x 10^{-2} N/m, respectively. How high will the mercury rise or be depressed in the tube as a result of capillary action?



If β > 90° then fluid level will be depressed

(A) -4.3 mm (depression)
(B) -1.6 mm (depression)
(C) 4.2 mm (rise)
(D) 6.4 mm (rise)

Plug & chug!

$h = \frac{4\sigma \cos\beta}{\rho d_{tube} g}$ ← from manual

Fluid Mechanics

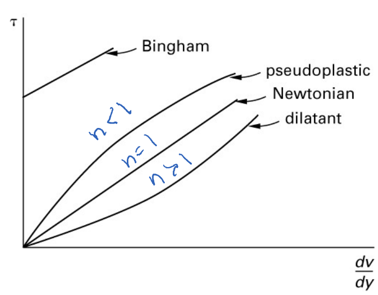
Stresses and Viscosity

Shear Stress

- Normal Component: $\tau_n = p = \frac{Force}{Area}$
- Tangential Component
  - For a Newtonian fluid: $\tau_t = \mu \frac{dv}{dy}$ (*v = velocity*)
  - For a pseudoplastic or dilatant fluid: $\tau_t = K\left(\frac{dv}{dy}\right)^n$

viscosity — rate of strain, or velocity gradient

Absolute Viscosity = μ = Ratio of shear stress to rate of shear deformation



Viscosity

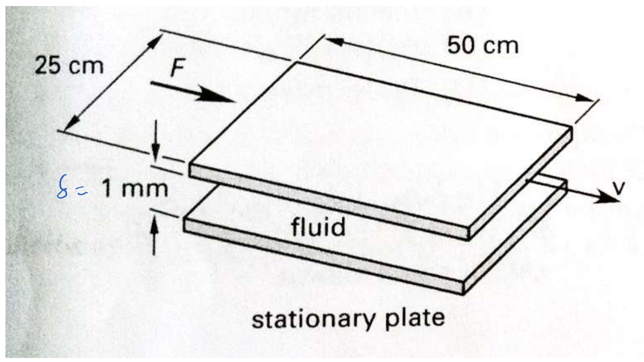
4. A sliding-plate viscometer is used to measure the viscosity of a Newtonian fluid. A force of 25 N is required to keep the top plate moving at a constant velocity of 5 m/s. What is the viscosity of the fluid?

$\tau = \mu \frac{dV}{dy} = \mu \left(\frac{V}{\delta}\right)$

$\tau = F/A$

$F = \frac{\mu V A}{\delta} \Rightarrow \mu = \frac{F\delta}{VA}$

(A) 0.005 N-s/m^2
(B) 0.04 N-s/m^2
(C) 0.2 N-s/m^2
(D) 5.0 N-s/m^2



Fluid Mechanics 9-2a1

Fluid Statics

Gage and Absolute Pressure

$p_{absolute} = p_{gage} + p_{atmospheric}$

Hydrostatic Pressure

$p = \gamma h + \rho g h$

$p_2 - p_1 = -\gamma(z_2 - z_1)$

Example (FEIM):
In which fluid is 700 kPa first achieved?

p_0 = 90 kPa

60 m	ethyl alcohol	p_1	7.586 kPa/m	$p_1 = p_0 + \gamma_{alc}(60\text{ m})$
10 m	oil	p_2	8.825 kPa/m	$p_2 = p_1 + \gamma_{oil}(10\text{ m})$
5 m	water	p_3	9.604 kPa/m	$p_3 = p_2 + \gamma_{H_2O}(5\text{ m})$
5 m	glycerin	p_4	12.125 kPa/m	$p_4 = p_3 + \gamma_{gly}(5\text{ m})$

(A) ethyl alcohol
(B) oil
(C) water
(D) glycerin

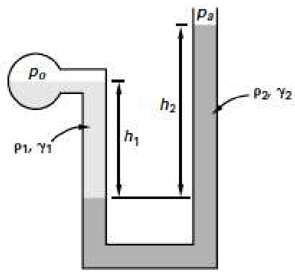
Ans: D

Fluid Statics

PRESSURE: $P_{absolute} = P_{gage} + P_{atmospheric} = P_{atmospheric} - P_{vacuum}$

HYDROSTATIC PRESSURE, $P = \gamma h$ * Pressure is ALWAYS NORMAL to a surface!

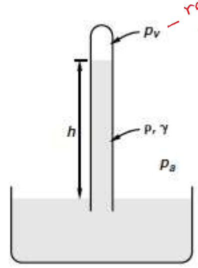
MANOMETRY: start w/ P on one end & add or subtract changes in hydrostatic pressure to fin P at other end ("TAKE A JOURNEY" through)



$$p_o - p_a = \gamma_2 h_2 - \gamma_1 h_1$$

Fluid Statics

Barometer



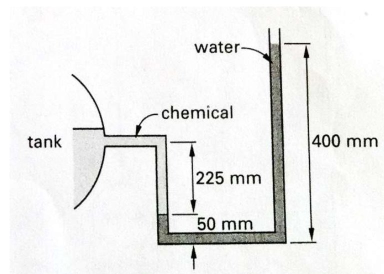
remember - this value affects the pressure. Don't neglect it!

Atmospheric Pressure

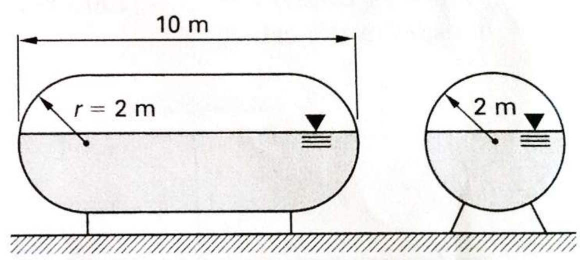
$$p_a - p_v = \rho g h \qquad \text{[SI]}$$

Manometry

5. An open water manometer is used to measure the pressure in a tank. The tank ifs half-filled with 50,000 kg of a liquid chemical that is not miscible in water. The manometer tube is filled with liquid chemical. What is the pressure in the tank relative to the atmospheric pressure?



(A) 1.4 kPa
(B) 1.9 kPa
(C) 2.4 kPa
(D) 3.4 kPa



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Manometry

1. Calc density of chemical using chemical mass & tank volume.

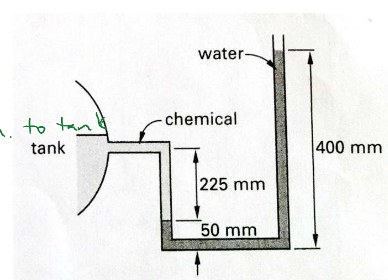
$$V = \overset{\text{sphere}}{\frac{4}{3}\pi r^3} + \overset{\text{cylinder}}{\pi r^2 (L - 2r)} = 108.7\ m^3$$

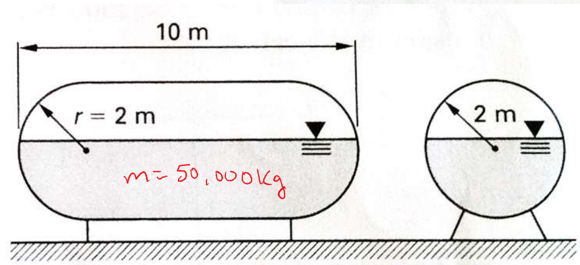
$$\rho = \frac{m}{Vol} = \frac{50000\ kg}{\frac{1}{2}(108.7\ m^3)} = 918.3\ kg/m^3$$

2. Use manometer to track pressure from atm. to tank

$$\overset{0,\ \text{gage}}{P_{atm}} + \gamma_{H_2O} h_{H_2O} - \overset{\gamma = \rho g}{\gamma_{chem}} h_{chem} = P_{tank}$$

$P_{tank} = 1407$ Pa





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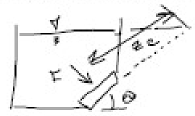
Fluid Statics

FORCES ON SUBMERGED SURFACES : FORCES ACT @ CENTER OF PRESSURE

F = VOLUME OF PRESSURE PRISM OR $F = \gamma A z_c \sin\theta$

LOCATION = CENTROID OF PRESSURE PRISM OR $y^* = \frac{I_{yz}}{z_c A}$; $z^* = \frac{I_{yy}}{z_c A}$



Volume

BUOYANCY : $F_{buoyant} = \gamma_{fluid} \forall_{displaced}$

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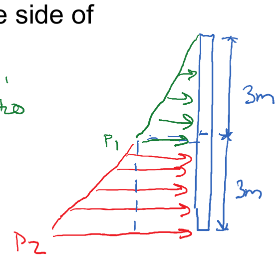
Hydrostatics

6. A 6m x 6m x 6m vented cubical tank is half-filled with water; the remaining space is filled with oil (SG=0.8). What is the total force on one side of the tank?

Since S.G.$_{oil}$ < S.G.$_{H_2O}$, the oil floats on the H_2O



(A) 690 kN
(B) 900 kN
(C) 950 kN
(D) 1.0 MN

Total Force = Volume of Pressure Prism

$P_1 = \rho_{oil}\, g \left(\frac{1}{2} h_{wall}\right) = 23544$ Pa

$P_2 = P_1 + \rho_{H_2O}\, g \left(\frac{1}{2} h_{wall}\right) = 52974$ Pa

$F = \frac{1}{2} P_1 \left(\frac{1}{2}h\right)(width) + \frac{1}{2} P_2 \left(\frac{1}{2}h\right)(width) + P_1 \left(\frac{1}{2}h\right)(width) = 9.01 \times 10^5$ N

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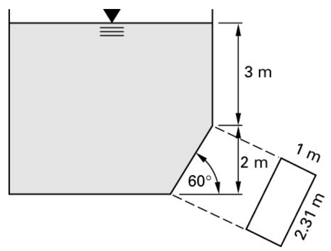
Fluid Statics Examples ~ FE likes these types of ?'s!

Example 1 (FEIM):
The tank shown is filled with water. What is the force that acts on a 1 m width of the inclined portion?



The surface under pressure is a rectangle 1 m at the base and 2.31 m tall.

Forces on Submerged Surfaces

$$R = pA \qquad 23.8$$

$$\overline{p} = \tfrac{1}{2}\rho g(h_1 + h_2) \qquad \text{[SI]} \qquad 23.10a$$

The average pressure on the inclined section is:

$$p_{ave} = \left(\tfrac{1}{2}\right)\left(997\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)(3\text{ m} + 5\text{ m})$$

$$= 39122\text{ Pa}$$

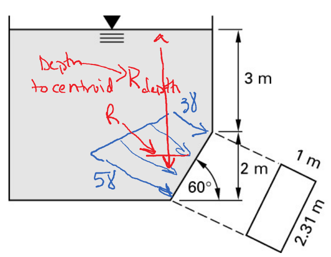
The resultant force is

$$R = p_{ave}A = (39122\text{ Pa})(2.31\text{ m})(1\text{ m})$$

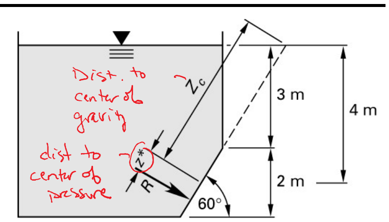
$$= 90372\text{ N}$$

Fluid Statics Examples

Example 1 (FEIM): (continued)
At what depth does the resultant force act?



The surface under pressure is a rectangle 1 m at the base and 2.31 m tall.



$$A = bh$$

moment of inertia of triangle →

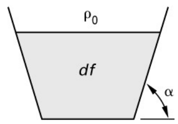
$$I_{y_c} = \frac{b^3h}{12}$$

$$Z_c = \frac{4\text{ m}}{\sin 60°} = 4.618\text{ m}$$

Fluid Mechanics 9-2e

Fluid Statics

Center of Pressure



$$y^* = \frac{\rho g I_{yz} \sin\alpha}{p_c A} \quad \text{[SI]} \quad \textit{23.17a}$$

$$z^* = \frac{\rho g I_{yy} \sin\alpha}{p_c A} \quad \text{[SI]} \quad \textit{23.18a}$$

If the surface is open to the atmosphere, then $p_0 = 0$ and

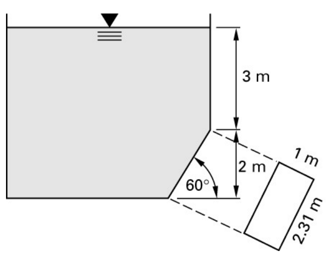
$$p_c = \overline{p} = \rho g z_c \sin\alpha \quad \text{[SI]} \quad \textit{23.19a}$$

$$y_{cp} - y_c = y^* = \frac{I_{yz}}{z_c A} \quad \textit{23.20}$$

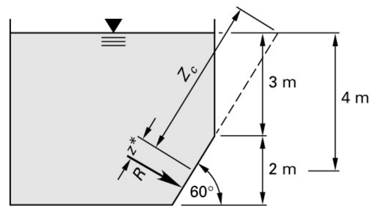
$$\boxed{z_{cp} - z_c = z^* = \frac{I_{yy}}{z_c A} \quad \textit{23.21}}$$

Fluid Statics Examples

Example 1 (FEIM):
At what depth does the resultant force act?



The surface under pressure is a rectangle 1 m at the base and 2.31 m tall.



$$A = bh$$

$$I_{y_c} = \frac{b^3 h}{12}$$

$$Z_c = \frac{4\ \text{m}}{\sin 60°} = 4.618\ \text{m}$$

$$z^* = \frac{I_{y_c}}{AZ_c} = \frac{b^3 h}{12 bhZ_c} = \frac{b^2}{12 Z_c}$$

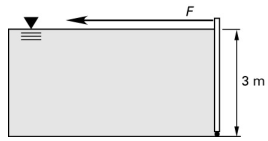
$$= \frac{(2.31\ \text{m})^2}{(12)(4.618\ \text{m})} = 0.0963\ \text{m}$$

$R_{depth} = (Z_c + z^*) \sin 60° = (4.618\ \text{m} + 0.0963\ \text{m}) \sin 60° = 4.08\ \text{m}$

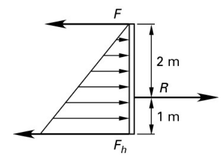
Fluid Statics Examples

Example 2 (FEIM):

The rectangular gate shown is 3 m high and has a frictionless hinge at the bottom. The fluid has a density of 1600 kg/m^3. The magnitude of the force F per meter of width to keep the gate closed is most nearly



(A) 0 kN/m
(B) 24 kN/m
(C) 71 kN/m
(D) 370 kN/m



$$p_{\text{ave}} = \rho g z_{\text{ave}} (1600 \frac{\text{kg}}{\text{m}^3})(9.81 \frac{\text{m}}{\text{s}^2})(\tfrac{1}{2})(3\text{ m})$$

$$= 23544 \text{ Pa}$$

$$\frac{R}{w} = p_{\text{ave}} h = (23544 \text{ Pa})(3 \text{ m}) = 70662 \text{ N/m}$$

$$F + F_h = R$$

R is one-third from the bottom (centroid of a triangle from the NCEES Handbook). Taking the moments about R,

$2F = F_h$

$$\frac{F}{w} = \left(\frac{1}{3}\right)\left(\frac{R}{w}\right) = \frac{70{,}667 \frac{\text{N}}{\text{m}}}{3} = 23.6 \text{ kN/m}$$

Therefore, (B) is correct.

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Hydrostatics

7. A gravity dam has the cross section shown. What is the magnitude of the resultant water force (per meter of width) acting on the face of the dam?

Horizontal Force = Volume of pressure prism
Vertical force = weight of water column above face

$F_x = \frac{1}{2} p_{max} \cdot A$ ← h · w $\Rightarrow \frac{F_x}{w} = \frac{1}{2}(490500 \text{ kPa})(50 \text{ m})$
$= 12.3 \times 10^6$ N/m

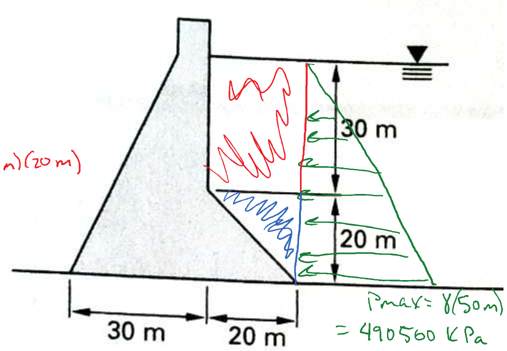
(A) 7.85 MN/m
(B) 12.3 MN/m
(C) 14.6 MN/m
(D) 20.2 MN/m

← Area · width

$F_y = \rho g V_{H_2O \text{ above}}$

$\frac{F_y}{w} = \rho g (A_{\square} + A_{\triangledown})$
$= 1000 \text{ kg/m}^3 (9.81 \text{ m/s}^2)[(20\text{ m})(30\text{ m}) + \frac{1}{2}(20\text{ m})(20\text{ m})]$
$= 7.85 \times 10^6$ N/m

Resultant $= \sqrt{R_x^2 + R_y^2}$
$= 14.6 \times 10^6$ N/m



$p_{max} = \gamma(50 \text{ m})$
$= 490500$ kPa

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Buoyancy

Archimedes' Principle and Buoyancy

- The buoyant force on a submerged or floating object is equal to the weight of the displaced fluid.
- A body floating at the interface between two fluids will have buoyant force equal to the weights of both fluids displaced.

$$F_{\text{buoyant}} = \gamma_{\text{water}} V_{\text{displaced}}$$

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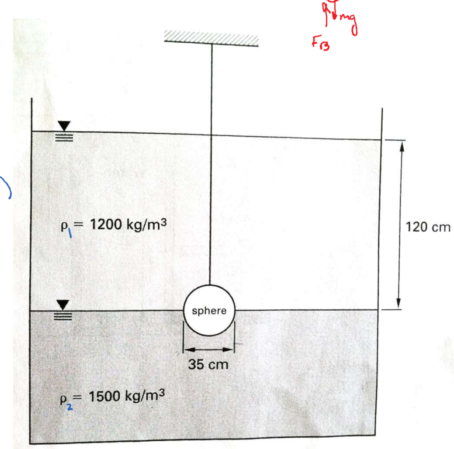
Buoyancy

8. A 35 cm diameter solid sphere (ρ = 4500 kg/m^3) is suspended by a cable as shown. Half of the sphere is in one fluid (ρ = 1200 kg/m^3) and the other half of the sphere is in another (ρ = 1500 kg/m^3). What is the tension in the cable?



F_B = wt of fluid displaced by the buoy

(A) 297 N

$F_B = \rho_1 g \left(\frac{1}{2} \forall_{sphere}\right) + \rho_2 g \left(\frac{1}{2} \forall_{sphere}\right)$

(B) 593 N

$= 297.3\ N$

(C) 694 N

use statics to find tension

(D) 826 N

$\Sigma F_y = 0$

$T - wt + F_B = 0$

$T = \rho_{sphere} \forall_{sphere} + F_B = 0$

$\Rightarrow T = 693.7\ N$

Fluid Dynamics

CONTINUITY: Mass is Conserved $\dot{m}_1 = \dot{m}_2$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

$$\rho_1 Q_1 = \rho_2 Q_2$$

For incompressible fluids (ρ is constant) $Q = A_1 v_1 = A_2 v_2 = \text{Etc.}$

BERNOULLI EQUATION: $\frac{P_1}{\gamma_1} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\gamma_2} + \frac{v_2^2}{2g} + z_2$ or $\frac{P_1}{\rho_1} + \frac{v_1^2}{2g} + g z_1 = \frac{P_2}{\rho_2} + \frac{v_2^2}{2g} + g z_2$

(might be called the field eqn.)

units = "HEAD" of liquid

REYNOLD'S NUMBER: $\mathrm{Re} = \frac{vD\rho}{\mu} = \frac{vD}{\nu}$
(Newtonian Fluids)

Laminar Flow: $\mathrm{Re} \lesssim 2100$

Turbulent Flow: $\mathrm{R} > 4000$

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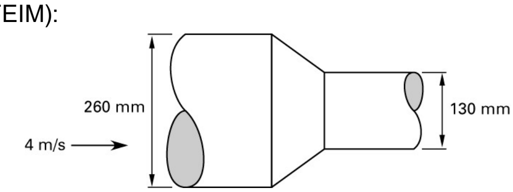
Continuity Equation Example

Example (FEIM):



The speed of an incompressible fluid is 4 m/s entering the 260 mm pipe. The speed in the 130 mm pipe is most nearly

(A) 1 m/s
(B) 2 m/s
(C) 4 m/s
(D) 16 m/s

$$A_1 v_1 = A_2 v_2$$

$$A_1 = 4A_2$$

so $v_2 = 4v_1 = (4)\left(4\ \frac{\text{m}}{\text{s}}\right) = 16\ \text{m/s}$

Therefore, (D) is correct.

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Bernoulli Equation

9. The diameter of a water pipe gradually changes from 5 cm at point A to 15 cm at point B. Point A is 5 m lower than point B. The pressure is 700 kPa at point A and 664 kPa at point B. Friction between the water and the pipe walls is negligible. What is the rate of discharge at point B?

$$\frac{p_A}{\gamma} + \frac{v_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{v_B^2}{2g} + z_B$$

rearrange: $v_B^2 = v_A^2 + \frac{2(p_A - p_B)}{\rho} + 2g(z_A - z_B)$

(A) 0.0035 m^3/s
(B) 0.0064 m^3/s
(C) 0.010 m^3/s
(D) 0.018 m^3/s

Relate velocities with continuity:

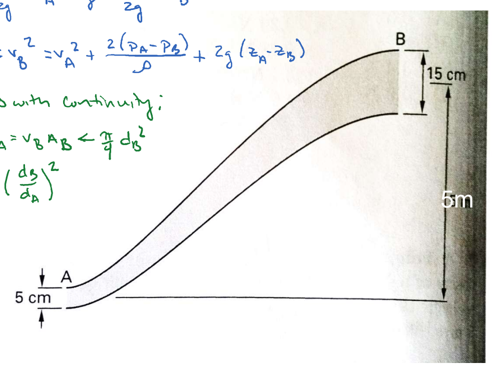
$Q_A = Q_B \Rightarrow v_A A_A = v_B A_B \leftarrow \frac{\pi}{4} d_B^2$

rearrange: $v_A = v_B \left(\frac{d_B}{d_A}\right)^2$

Combine & solve

$v_B = 0.571$ m/s

$Q = v_B A_B = 0.01 \, m^3/s$



Bernoulli – Flow from a Jet

$\gamma = 0.9\gamma_{H_2O}$

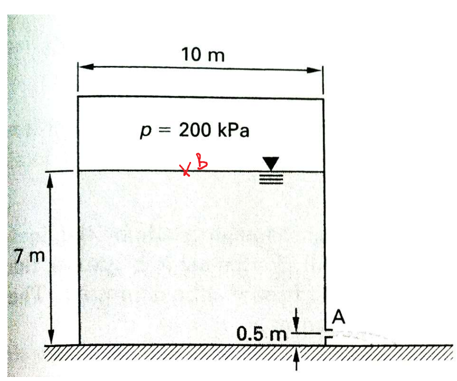
10. A liquid with a specific gravity of 0.9 is stored in a pressurized, closed storage tank. The tank is cylindrical with a 10 m diameter. The absolute pressure in the tank above the liquid is 200 kPa. What is the initial velocity of a fluid jet when a 5 cm diameter orifice is opened at point A?

(A) 11.3 m/s
(B) 18.0 m/s
(C) 18.6 m/s
(D) 23.9 m/s

200 kPa, abs ≈0 101.3 kPa, abs

$$\frac{p_B}{\gamma} + \cancel{\frac{v_B^2}{2g}} + z_B = \frac{p_A}{\gamma} + \frac{v_A^2}{2g} + z_A$$

solve for $v_A \rightarrow v_A = 18.6$ m/s



Flow in Pipes – Fluid Dynamics and Friction

Steady, Incompressible Flow

ENERGY EQUATION: $\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + \underbrace{h_f}_{\text{friction loss}} + h_L \leftarrow$ loss due to fittings

$\rightarrow h_{L,\text{fitting}} = C\left(\frac{v^2}{2g}\right) \leftarrow$ velocity through fitting

$\rightarrow h_f = f\frac{L}{D}\frac{v^2}{2g}$; f is a function of Re and ϵ/D ; ϵ = surface roughness

LAMINAR FLOW: $f = 64/\text{Re}$ & $h_f = \frac{32\mu L v}{\gamma D^2}$

TURBULENT FLOW: use MOODY DIAGRAM to find f

Smooth & Hydraulically smooth pipes: $f = \frac{0.316}{\text{Re}^{1/4}}$ (for Re < 100,000)

Can Also use HAALAND EQN FOR ALL TURBULENT FLOW

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\frac{6.9}{\text{Re}} + \left(\frac{\epsilon/D}{3.7}\right)^{1.11}\right]$$

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Reynolds Number

For a Newtonian fluid:

$$\text{Re} = \frac{\text{v}D\rho}{\mu} \quad \text{[SI]}$$

$$\text{Re} = \frac{\text{v}D}{\nu}$$

D = hydraulic diameter = $4R_H$

ν = kinematic viscosity

μ = dynamic viscosity

For a pseudoplastic or dilatant fluid:

$$\text{Re}' = \frac{\text{v}^{2-n}D^n\rho}{K\left(\frac{3n+1}{4n}\right)^n 8^{n-1}}$$

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Fluid Dynamics and Friction

11. A steel pipe with an inside diameter of 25 mm is 20 m long and carries water at a rate of 4.5 m^3/h. Assuming the specific roughness of the pipe is 0.00005 m, the water has an absolute viscosity of 1.00×10^{-3} Pa-s and a density of 1000 kg/m^3, what is the friction factor?

(A) 0.023
(B) 0.030
(C) 0.026
(D) 0.028

- Continuity to find v

  $v = \frac{Q}{A} = 2.55$ m/s

- Find Re ← Reynold's Number

  $Re = \frac{\rho v D}{\mu} = 6.4 \times 10^4$

- Find ε/D ← relative roughness

  $\varepsilon/D = 0.002$

- Use Moody diagram to approximate

(In my opinion, the choices given for answers are unfair. The moody diagram is not that precise!)

Fluid Mechanics

Head Loss in Conduits and Pipes

Minor Losses in Fittings, Contractions, and Expansions

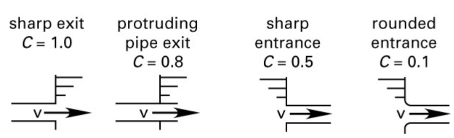
- Bernoulli equation + loss due to fittings in the line and contractions or expansions in the flow area

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_f + h_{L,\text{fitting}} \quad \text{[U.S.]} \qquad 24.30b$$

$$h_{L,\text{fitting}} = C\left(\frac{v^2}{2g}\right) \qquad 24.31$$

Entrance and Exit Losses

- When entering or exiting a pipe, there will be pressure head loss described by the following loss coefficients:



Non-Circular Conduits, Open Channel Flow, and Partially Full Pipes

Hydraulic Radius, $R_H = \frac{\text{area in flow}}{\text{wetted perimeter}}$; Equivalent Diameter, $D_H = 4R_H$

OPEN CHANNEL: $v = \left(\frac{1}{n}\right) R_H^{2/3} S^{1/2}$ (SI UNITS) n = Manning's Roughness coefficient

$v = \left(\frac{1.486}{n}\right) R_H^{2/3} S^{1/2}$ (U.S. UNITS) S = Slope of channel

PARTIALLY FULL PIPES

$v = 0.849\, C R_H^{0.63} S^{0.54}$ (SI) C = Hazen-Williams Roughness Coeff.

$v = 1.318\, C R_H^{0.63} S^{0.54}$ (US)

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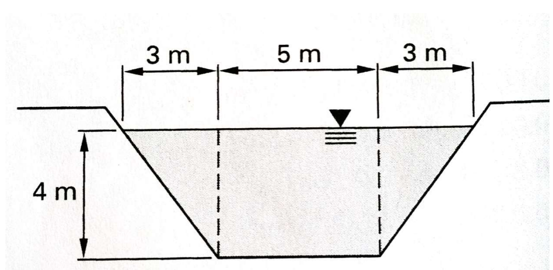
Hydraulic Radius

12. What is the hydraulic radius of the trapezoidal irrigation canal shown?

$R_H = \frac{\text{cross-sectional area of flow}}{\text{wetted perimeter}}$

$R_H = \frac{32\text{ m}^2}{15\text{ m}} = 2.13\text{ m}$

(A) 1.63 m
(B) 2.00 m
(C) 2.13 m
(D) 4.00 m



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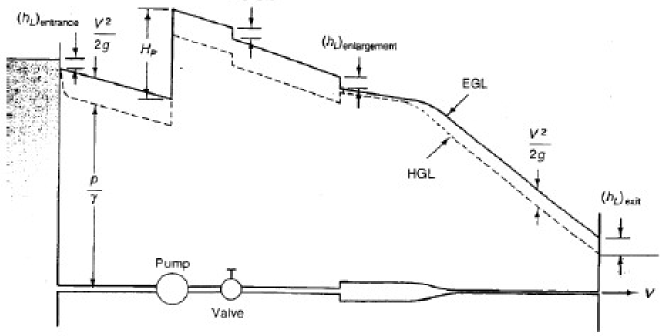
Hydraulic Grade Line and Energy Grade Line Pump Power

HYDRAULIC GRADE LINE & ENERGY GRADE LINE
(HGL) (EGL)



NOTE:
$EGL = HGL + \frac{v^2}{2g}$

. Hydraulic grade line (HGL) and energy grade line (EGL) for a piping system.

PUMP POWER, $P = \dot{W} = \frac{Q\gamma h_p}{\eta}$ ← h_p = head added by pump to the fluid
↖ pump efficiency

Express in Watts (1 N·m/s) or horsepower (550 ft·lb/s)

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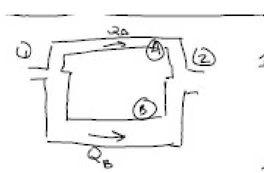
Multi-path Pipelines



3 MAIN PRINCIPLES

1) Head Loss in each branch is equal : $h_{f,A} = h_{f,B}$

$$f_A \frac{L_A}{D_A} \frac{v_A^2}{2g} = f_B \frac{L_B}{D_B} \frac{v_B^2}{2g}$$

2) Head Loss between the 2 junctions is same as head loss in each branch

3) Mass Must be conserved. Therefore the total flow rate equals the sum of the flow rate in each branch.

$$Q_1 = Q_2 = Q_A + Q_B \quad \text{OR} \quad \frac{\pi}{4} D_1^2 v_1 = \frac{\pi}{4} D_2^2 v_2 = \frac{\pi}{4} D_A^2 v_A + \frac{\pi}{4} D_B^2 v_B$$

THIS SIMPLIFIES TO: $D_1^2 v_1 = D_2^2 v_2 = D_A^2 v_A + D_B^2 v_B$

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Multi-path pipelines

13. The Darcy friction factor for both of the pipes ← $f_1 = f_2$
shown is 0.024. The total flow rate is 300 m^3/h.
What is the flow rate through the 250 mm pipe?

we know for parallel pipes, $h_{f,1} = h_{f,2}$; $Q_1 + Q_2 = Q_{total}$

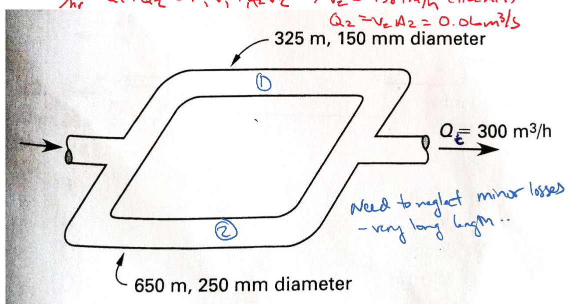
$$h_{f_1} = h_{f_2} \Rightarrow \cancel{f_1} \frac{L_1}{D_1} \frac{v_1^2}{\cancel{2g}} = \cancel{f_2} \frac{L_2}{D_2} \frac{v_2^2}{\cancel{2g}} \Rightarrow \frac{L_1}{D_1} v_1^2 = \frac{L_1}{D_2} V_2$$

$$\Rightarrow v_1 = 1.075\, v_2$$

(A) 0.04 m^3/s
(B) 0.05 m^3/s
(C) 0.06 m^3/s
(D) 0.07 m^3/s

$$Q_{total} = 300\ m^3/hr = Q_1 + Q_2 = A_1 V_1 + A_2 V_2 \Rightarrow v_2 = 4381\ m/h\ (1.22\ m/s)$$

$$Q_2 = V_2 A_2 = 0.06\ m^3/s$$



Need to neglect minor losses
– very long length..

Implulse-Momentum

IMPULSE-MOMENTUM

$$\Sigma F = \rho Q (V_2 - V_1) = \left(\frac{Q\gamma}{g} V\right)_{out} - \left(\frac{Q\gamma}{g} V\right)_{in}$$

BE SURE TO DRAW FBD!
WATCH YOUR SIGNS!

Be sure to familiarize yourself with how this equation works for bends, enlargements, contractions, jet propulsion, fixed blades, moving blades, and impulse turbines.

Impulse-Momentum

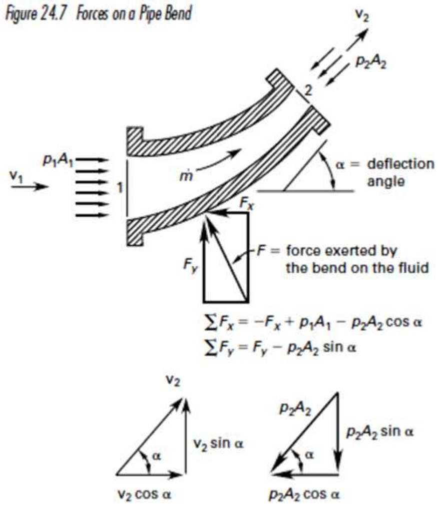
$$\sum \mathbf{F} = Q_2\rho_2\mathbf{v}_2 - Q_1\rho_1\mathbf{v}_1 \quad \text{[SI]} \qquad 24.38a$$

Pipe Bends, Enlargements, and Contractions

$$-F_x = p_2A_2\cos\alpha - p_1A_1 + Q\rho(\mathrm{v}_2\cos\alpha - \mathrm{v}_1) \quad \text{[SI]} \qquad 24.39a$$

$$F_y = (p_2A_2 + Q\rho\mathrm{v}_2)\sin\alpha + m_{\text{fluid}}g \quad \text{[SI]} \qquad 24.40a$$

Figure 24.7 Forces on a Pipe Bend



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Fluid Mechanics 9-6a

Impulse-Momentum Principle

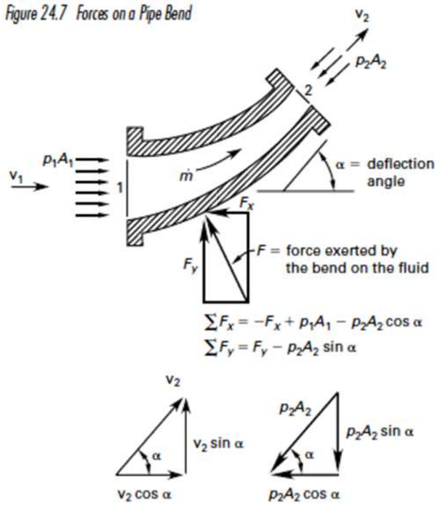
$$\sum \mathbf{F} = Q_2\rho_2\mathbf{v}_2 - Q_1\rho_1\mathbf{v}_1 \quad \text{[SI]} \qquad 24.38a$$

Pipe Bends, Enlargements, and Contractions

$$-F_x = p_2A_2\cos\alpha - p_1A_1 + Q\rho(\mathrm{v}_2\cos\alpha - \mathrm{v}_1) \quad \text{[SI]} \qquad 24.39a$$

$$F_y = (p_2A_2 + Q\rho\mathrm{v}_2)\sin\alpha + m_{\text{fluid}}g \quad \text{[SI]} \qquad 24.40a$$

Figure 24.7 Forces on a Pipe Bend



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Fluid Mechanics 9-6b1

Impulse-Momentum Principle

Example (FEIM):
Water at 15.5˚C, 275 kPa, and 997 kg/m^3 enters a 0.3 m × 0.2 m reducing elbow at 3 m/s and is turned through 30˚. The elevation of the water is increased by 1 m. What is the resultant force exerted on the water by the elbow? Ignore the weight of the water.

Ans: 13118 N

Fluid Mechanics 9-7a

Impulse-Momentum Principle

Initial Jet Velocity: $\mathrm{v} = \sqrt{2gh}$ *24.41*

Jet Propulsion:

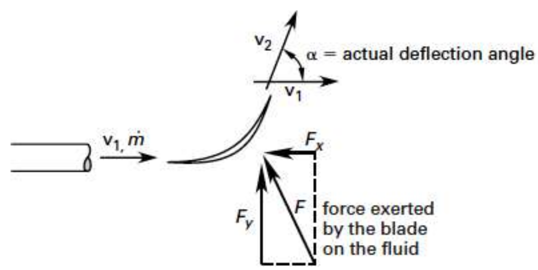
$$\begin{aligned} F &= \dot{m}(\mathrm{v}_2 - \mathrm{v}_1) \\ &= \dot{m}(\mathrm{v}_2 - 0) \\ &= Q\rho\mathrm{v}_2 \\ &= \mathrm{v}_2 A_2 \rho \mathrm{v}_2 \\ &= A_2 \rho \mathrm{v}_2^2 \\ &= A_2 \rho \left(\sqrt{2gh}\right)^2 \\ &= 2g\rho h A_2 \\ &= 2\gamma h A_2 \end{aligned}$$ *24.42*

Fluid Mechanics 9-7b1

Impulse-Momentum Principle

Fixed Blades

Figure 24.9 Open Jet on a Stationary Blade



$$-F_x = Q\rho(\mathrm{v}_2 \cos\alpha - \mathrm{v}_1) \quad \text{[SI]} \qquad \textit{24.43a}$$

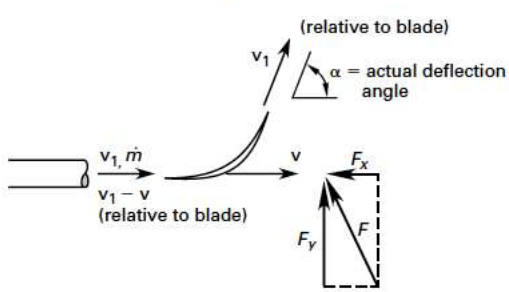
$$F_y = Q\rho \mathrm{v}_2 \sin\alpha \quad \text{[SI]} \qquad \textit{24.44a}$$

Fluid Mechanics 9-7b2

Impulse-Momentum Principle

Moving Blades

Figure 24.10 Open Jet on a Moving Blade



$$-F_x = -Q\rho(\mathrm{v}_1 - \mathrm{v})(1 - \cos\alpha) \quad \text{[SI]} \qquad \textit{24.45a}$$

$$F_y = Q\rho(\mathrm{v}_1 - \mathrm{v})\sin\alpha \quad \text{[SI]} \qquad \textit{24.46a}$$

Fluid Mechanics 9-7c

Impulse-Momentum Principle

Impulse Turbine

Figure 24.11 Impulse Turbine

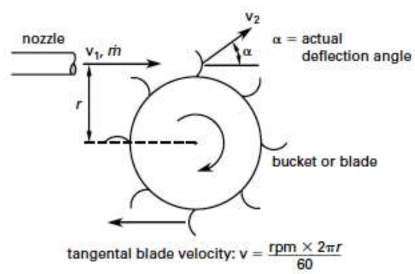
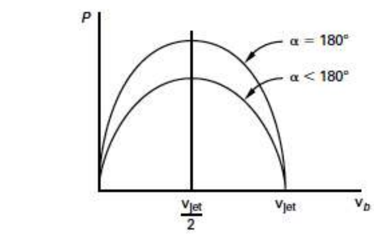


Figure 24.12 Turbine Power



$$P = Q\rho(v_1 - v)(1 - \cos\alpha)v \quad \text{[SI]} \quad 24.47a$$

The maximum power possible is the kinetic energy in the flow.

$$P_{max} = \frac{Q\rho v_1^2}{2} \quad \text{[SI]} \quad 24.49a \qquad P_{max} = \frac{Q\gamma v_1^2}{2g} \quad \text{[U.S.]} \quad 24.49b$$

The maximum power transferred to the turbine is the component in the direction of the flow.

$$P_{max} = Q\rho\left(\frac{v_1^2}{4}\right)(1 - \cos\alpha) \quad \text{[SI]} \quad 24.48a$$

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Impulse-Momentum

14. Water is flowing at 50 m/s through a 15 cm diameter pipe. The pipe makes a 90 degree bend, as shown. What is the reaction on the water in the z-direction at the bend?

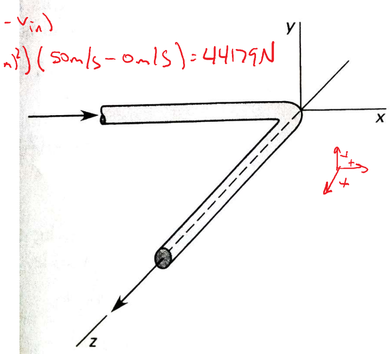
* Find Force on the fluid due to its motion

(A) -44 kN
(B) -33 kN
(C) 14 kN
(D) 44 kN

$\Sigma F_z = Q_2 \rho_2 v_2 - Q_1 \rho_1 v_1$

$\Sigma F_z = Q\rho(v_{out} - v_{in})$

$\Sigma F_z = 50 m/s \left(\frac{\pi}{4}(.15m)^2\right)(50m/s - 0m/s) = 44179N$



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Flow and Pressure Measurement

PITOT TUBE – USED TO MEASURE FLOW VELOCITY $V = \sqrt{\frac{2(p_o - p_s)}{\rho}} = \sqrt{\frac{2g(p_o - p_s)}{\gamma}}$

p_0 = static pressure ; p_s = stagnation pressure
also called impact pressure

VENTURI & ORIFICE METERS
(See sketches in Ref. Book for variable descriptions)

Venturi

$$Q = \left(\frac{C_v A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \right) \sqrt{2\left(\frac{p_1}{\rho} + g z_1 - \frac{p_2}{\rho} - g z_2\right)} \quad (SI)$$

$$Q = \left(\frac{C_v A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \right) \sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2\right)} \quad (US)$$

Orifices

$$Q = CA\sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2\right)} \quad (U.S.)$$

$$Q = CA\sqrt{2\left(\frac{p_1}{\rho} + g z_1 - \frac{p}{\rho} - g z_2\right)} \quad (S.I.)$$

Submerged Orifices

$$Q = CA\sqrt{2g(h_1 - h_2)}$$

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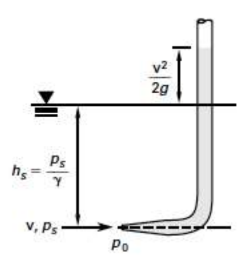
Fluid Mechanics
Fluid Measurements

Pitot Tube – measures flow velocity

Figure 25.1 Pitot Tube



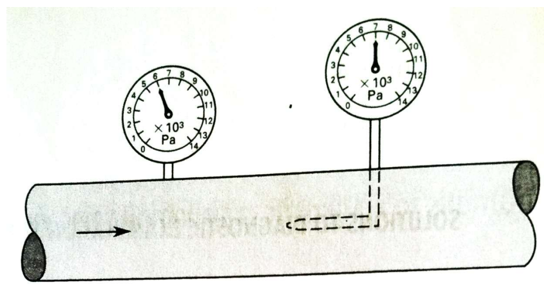
- The static pressure of the fluid at the depth of the pitot tube (p_0) must be known. For incompressible fluids and compressible fluids with M ≤ 0.3,

$$v = \sqrt{\frac{2(p_0 - p_s)}{\rho}} \quad [SI] \quad 25.11a$$

Flow and Pressure Measurement

15. The density of air flowing in a duct is 1.15 kg/m^3. A pitot tube is placed in the duct as shown. The static pressure in the duct is measured with a wall tap and pressure gage. Use the gage readings to determine the velocity of the air.

(A) 42 m/s
(B) 102 m/s
(C) 110 m/s
(D) 150 m/s



use formula from manual on previous slide

$v = \sqrt{\frac{2(p_0 - p_s)}{\rho}}$

$= 4.17$ m/s

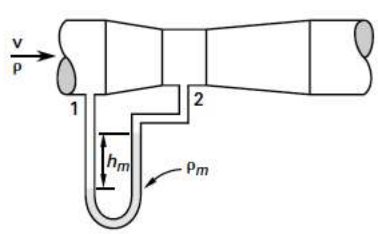
Fluid Mechanics

Fluid Measurements

Venturi Meters – measures the flow rate in a pipe system

- The changes in pressure and elevation determine the flow rate. In this diagram, $z_1 = z_2$, so there is no change in height.

Figure 25.2 Venturi Meter with Differential Manometer



$$Q = \left(\frac{C_v A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}}\right)\sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2\right)}$$

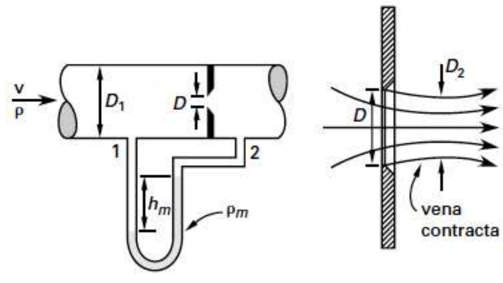
[U.S.] 25.14b

Fluid Mechanics

Fluid Measurements

Orifices

Figure 25.3 Orifice Meter with Differential Manometer



(a) orifice meter (b) orifice plate

$$Q = CA\sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2\right)} \quad \text{[U.S.]} \quad 25.17b$$

Orifice Flow

16. Water flows out of a tank at 12.5 m/s from an orifice located 9m below the surface. The cross-sectional area of the orifice is 0.002 m^2, and the coefficient of discharge is 0.85. What is the diameter D, at the vena contracta?

coefficient of contraction

$A_{\text{vena contracta}} = c_c A_{\text{opening}}$

$c_c = \frac{C}{c_v}$ ← coeff. of discharge ← coeff. of velocity

$c_v = \frac{v}{v_{\text{ideal @ that depth}}} = \frac{v}{\sqrt{2gh}}$

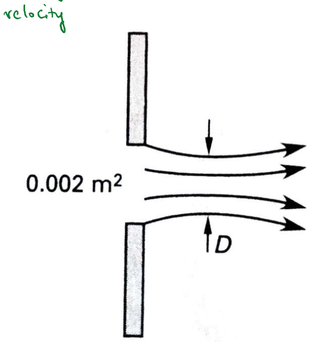
(A) 4.2 cm
(B) 4.5 cm
(C) 4.7 cm
(D) 4.8 cm

$c_v = 0.941$

$c_c = 0.903$

$A_{vc} = 0.00181 m^2$

$d = 4.8 cm$



Similitude

Geometric Similarity : Model is true to length, area & volume
Kinematic Similarity : Flow Regimes of model & prototype are the same
Dynamic Similarity : Ratios of all types of forces are equal for model + prototype

Met if following simultaneous equations are satisfied for model & prototype

$$\left[\frac{\rho v^2}{p}\right]_m = \left[\frac{\rho v^2}{p}\right]_p \; ; \; Re_m = Re_p \; ; \; Fr_m = Fr_p \; ; \; Ca_m = Ca_p \; ; \; We_p = We_m$$

$$Re = \frac{v l \rho}{\mu} \; ; \; Fr = \frac{v^2}{lg} \; ; \; C_a = \frac{\rho v^2}{E} \; ; \; We = \frac{\rho l v^2}{\sigma}$$

E: modulus of elasticity; σ: surface tension

In other words… Dimensionless parameters must be equal between the model and prototype
(Each dimensionless parameter is a ratio of different types of forces being exerted on the fluid)

For example:
For completely submerged models/prototypes and pipe flow, the Reynolds (Re) numbers must be equal
For Weirs, dams, ships, and open channels, the Froude (Fr) numbers must be equal

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Similitude – Dimensionless Parameters

Reynolds number =	$\frac{\text{inertial force}}{\text{viscous force}}$	$Re = \frac{V\ell\rho}{\mu}$
Froude number =	$\frac{\text{inertial force}}{\text{gravity force}}$	$Fr = \frac{V^2}{\ell g}$
Mach number =	$\frac{\text{inertial force}}{\text{compressibility force}}$	$M = \frac{V}{c}$
Weber number =	$\frac{\text{inertial force}}{\text{surface tension force}}$	$We = \frac{V^2\ell\rho}{\sigma}$
Strouhal number =	$\frac{\text{centrifugal force}}{\text{inertial force}}$	$St = \frac{\ell\omega}{V}$
Pressure coefficient =	$\frac{\text{pressure force}}{\text{inertial force}}$	$C_p = \frac{\Delta p}{\frac{1}{2}\rho V^2}$
Drag coefficient =	$\frac{\text{drag force}}{\text{inertial force}}$	$C_D = \frac{\text{drag}}{\frac{1}{2}\rho V^2 A}$

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Similitude

17. A nuclear submarine is capable of a top underwater speed of 65 km/h. How fast would a 1/20 scale model of the submarine have to be moved through a testing pool filled with seawater for the forces on the submarine and model to be dimensionally similar?

Submerged, so $Re_{model} = Re_{prototype}$

(A) 0.90 m/s
(B) 18 m/s
(C) 180 m/s
(D) 360 m/s

$$\left(\frac{\rho v l}{\mu}\right)_m = \left(\frac{\rho v l}{\mu}\right)_p \qquad \mu_m = \mu_p \quad \rho_m = \rho_p$$

$\Rightarrow V_{model} = 361.1\ m/s$

Drag

$F_D = \frac{1}{2} C_D A \rho v^2$; For Laminar Flow, $F_D = 24/Re$ (For $Re < 0.1$)

Typically these problems will be "plug and chug"

Drag Coefficients for Spheres and Circular Flat Disks

Figure 24.14 Drag Coefficients for Spheres and Circular Flat Disks